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# An Imperfect Preventive Maintenance Policy Considering Reliability Limit

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**Abstract**
This paper discusses an imperfect preventive maintenance model for a deteriorating repairable system with consideration of the reliability limitand random maintenance quality. The model is derived from the combination of failure rate adjustment and age over an infinite time horizon. The maintenance intervals are obtained assuming both the failure rate increase factor and age restoration factor are random variables with a uniform distribution. The optimal policy with a sensitivity analysis showing how different cost parameters affect the long run average maintenance cost rate is presented.
**Keywords:** Imperfect preventive maintenance, virtual age, failure rate, reliability limit,

## I. Introduction

The maintenance of deteriorating systems is often imperfect. Previous studies have shown that the imperfect preventive maintenance (PM) can reduce the wear rate and aging effects of deteriorating systems to a certain level between the conditions of '*as good as new'* (AGAN) and '*as bad as old'* *(*ABAO),and partially restore the performance and reliability of the system so as to extend its life and reduce the frequency of failures[1]. This is achieved by the two commonly used policies, periodic and sequential preventive maintenance actions. In order to satisfy the requirements of high reliability and low maintenance cost for aging equipment during the wear period, a suitable sequential PM policy is necessary.

Maintenance optimization of repairable systems was initiated by [2]. Since then, a number of maintenance models for repairable systems have appeared in literature.[3]proposed the improvement factor method to measure the restoration effect of PM for the deteriorating systems, two maintenance policies, i.e., single and multi-component systems. And [4] proposed sequential imperfect PM model and two sequential PM models called hazard rate adjustment and age models were proposed by [5].

[6]and[7] introduced adjustment factors in hazard rate and effective age after imperfect PM models respectively. [8] extended model [2]into two maintenance policies, i.e., single and multi-component systems and observed that, the effect of maintenance actions can be modeled using system virtual age or failure rate functions.[9,10] introduced the concept of virtual age to model the effect of imperfect repair. [11] studied the modelling of the hazard rate restoration after performing a PM activity. These models are to determine the optimal time interval between PMs and the number of PMs before replacing the system by minimizing the expected average cost over a finite or infinite time span.[12] generalized the virtual age model to a virtual age process for imperfect repair of repairable systems and [13] extended the age reduction idea to model periodic imperfect PM. [14,15] introduced two hybrid sequential PM models incorporating failure rate adjustment and age reduction factors. The two models assumed that PM actions not only reduce the effective age to a certain value but also increase the slope of failure rate function as equipment ages.

Based on the reduction of failure intensity and virtual age,[16] proposed two classes of imperfect repair models based on the reduction of failure intensity and virtual age, which is arithmetic reduction of intensity (ARI) model and arithmetic reduction of age (ARA) model, respectively.

[17] proposed a general PM model that incorporates three types of PM, i.e., imperfect preventive maintenance(IPM), perfect preventive maintenance (PPM) and failed preventive maintenance (FPM), and obtained an optimal PM schedule maximizing the availability of a repairable system.[18]considered sequential PM model for a finite time span. They considered three models: minimal repair, block replacement and simple replacement.[19] built on [20] model where each PM application reduces the effective age. They considered age reduction model where parts of the system are non-maintainable. [21] introduced phasic sequential PM that considers improvement factor for machine age.[22]considered stochastic maintenance policy for a system degradation over a finite life time. They usedgenetic algorithm (GA) to solve their model. The maintenance improvement was considered stochastic. [23] considered two types of failures, catastrophic and

minor, and considered the improvement in hazard rate function.[24]did a comparative analysis of construction equipment failures using the classical power law models and the new time series models; the researcher found out that the power law models are easy to apply and are capable of predicting reliability metrics at both the system and subsystem levels with fair results, while time series models based on predictive data mining algorithms are more flexible, comprehensive, and accurate by taking various influencing factors into account.Although the primary criterion for judging preventive maintenance is increasing in availability, most of the existing optimal sequential PM policies are developed by minimizing the expected cost rate only without accounting for reliability limit.

This makes the system to have a very low reliability at the time of preventive replacement or overhaul,[25],[26],[27]because in practicehigh reliability is usually required to avoid high probability of system unexpected failures[28], [29]. [30]observed that, each PM action in sequential PM model has a different quality, which requires a large number of failure data to estimate it. However, in real circumstances, it is usually difficult to specifythe quality of a PM action precisely as it depends on the available resources, i.e., material, technological, human, time, etc. Some researcher seven assumed that the quality of PM as a fixed constant, which is usually not true in many situations. In this case, it is suitable to assume that restoration factor of PM or failure rate increase factor is a random variable with a probability distribution[30].

This article analyses a hybrid model of sequential imperfect PM by minimizing the long-run average cost rate considering reliability limit for deteriorating repairable systems with random maintenance quality. The proposed model regards the failure rate increase factor and the restoration factor as random variables with a uniform probability distribution. An example is also presented with a discussion of parameters that affect maintenance quality.

## II. Model Description and Assumptions

a. The repairable system deteriorates with usage and age. The planning horizon is infinite and the times for preventive maintenance, minimal repair and replacement is negligible.
b. The system failure rate without PM is continuous and strictly increasing. The failure rate after the $i_{th}$PM is $h_i(x) = \theta h_{i-1}(x + qt_{i-1}), 0 \leq x < T_i$
c. The system undergoes minimal repair upon failures between two adjacent PM actions and the repairs do not change the failure rate or the system effective age.
d. The imperfect PM is performed at a sequence $t_1, t_2, \ldots, T_{N-1}$, and the system is replaced at $T_N$ to '*as good as new'*$(AGAN)$state.
e. The failure rate increase factor and restoration factor are all random variables and follow uniform distribution.

Notation

*f(.)* probability density function *(pdf)*
*F(.)* cumulative distribution function *(cdf)*
*Fi* number of failures for the $i_{th}$ PM interval
*h(.)* system failure rate
$n$ number of failures for an observed repairable system
$N$ number of PM actions before system replacement
$N^*$ optimal value of $N$
$q$ restoration factor immediately after the $i_{th}$ PM, $0 \leq q \leq 1$
*R* predetermined reliability
*R(.)* reliability function
$t_i$ time for the $i_{th}$ PM, $t_1 + t_2 +, \ldots, T_N$ $(i = 1,2, \ldots, N)$
$T_i$ time interval between $(i\text{-}1)_{th}$ and $i_{th}$ PM actions $(i = 1,2, \ldots, N)$
$u$ failure rate increase factor upper limit
$\theta$ failure rate increase factor immediately after the $i_{th}$ PM, $u \geq \theta \geq 1$
$v_i$ system virtual age after $i_{th}$PM $(i = 1,2,\ldots,N$
β Weibull shape parameter *(β > 1)*
λ Weibull scale parameter *(λ > 0)* $c_m$, $c_p$, $c_r$ minimal repair, PM and system replacement cost respectively.
*C(N)*, *C(N*)*long run average cost rate with $N$ and $N^*$ cycles respectively.

### 2.1 The failure rate and reliability function of the model

Consider a repairable system which is put into operation at time $t = 0$ and is minimally repaired between two adjacent preventive maintenance (PM) actions upon failure. Based on Kijima type virtual age idea, the virtual age $\vartheta_i$ of the equipment after the $i_{th}$ PM can be given by [9].

$$\vartheta_i = \vartheta_{i-1} + qT_i = qt_i (i = 1,2, \ldots, N) \quad (1)$$

When $q = 0$ , it implies the equipment has been restored to *'as good as new'* (AGAN) state, i.e., perfect maintenance has been performed; while $q = 1$ means the maintenance action has no effect on the condition of the equipment and it remains *'as bad as old'*(ABAO),i.e., minimal repair or imperfect maintenance has been performed.

According to[9], if the equipment's virtual age $\vartheta_{i-1}$ after the $(i\text{-}1)_{th}$ PM, then the time interval $x$ between the $(i\text{-}1)_{th}$ and $i_{th}$ PM has the following conditional cumulative distribution function *(cdf)*

$$F(x|\vartheta_{i-1}) = \frac{F(x+\vartheta_{i-1})-F(\vartheta_{i-1})}{1-F(\vartheta_{i-1})} = 1 - \frac{R(x+\vartheta_{i-1})}{R(\vartheta_{i-1})}, 0 \le x \le T_i \quad (2)$$

where $F(\cdot)$ and $R(\cdot)$ are *cdf* and reliability functions respectively.

The reliability function of the Weibull process for the interval *x* can be expressed as [31-33]

$$R(x) = exp\left(-\lambda x^{\beta}\right), 0 \le x \le T_i \quad (3)$$

where $\lambda(\lambda > 0)$ and $\beta(\beta > 1)$ are the scale and shape parameters of the Weibull process respectively. Adopting the Weibull process, being the widely used in Kijima type virtual age model for repairable systems due to its flexibility, to describe the imperfect maintenance based on equations (1) to (3).The conditional reliability of the equipment within the time interval can be derived as follows

$R(x|\vartheta_{i-1}) = \frac{R(x+\vartheta_{i-1})}{R(\vartheta_{i-1})} = exp\{-\lambda[(x +$ *qti− 1β− qti− 1β0≤x≤Ti* (4)

Taking the negative derivative with respect to *x* in equation (4), we get the corresponding probability density function *(pdf)*

$f(x|\vartheta_{i-1}) = \frac{-dR(x|\vartheta_{i-1})}{dx} = \lambda\beta(x +$ *qti− 1β− 1*exp*−λx+qti− 1β− qti− 1β*

$0 \le x \le T_i$ (5)

Thus, the conditional failure rate can be obtained by

$$h(x|\vartheta_{i-1}) = \frac{f(x|\vartheta_{i-1})}{R(x|\vartheta_{i-1})} = \lambda\beta(x + qt_{i-1})^{\beta-1}, 0 \le x \le T_i \quad (6)$$

The failure rate of deteriorating repairable systems normally increases with usage and age especially during the wear period. Therefore, the system would need more frequent maintenance actions as the failure rate of the $i_{th}$ PM interval increases more quickly than that of the $(i\text{-}1)_{th}$ PM interval. For the proposed hybrid model, the failure rate increase factor $\theta$ is added to the Kijima type virtual age model to determine the maintenance policy.

Assuming that the failure rate of a deteriorating repairable system after the $i_{th}$PM is: $h_i(x) = \theta h_i(x + qt_{i-1})$ , then $h_i(x) = \theta^{i-1} h(x + qt_{i-1})$, $where\ \theta(\theta \ge 1)$ is the failure rate increase factor and is a random variable, and $h_{(x)}$ is the equipment failure rate when there are no PM actions. Therefore, from equations (4) and (6), the combined reliability and failure rate function of the proposed model within the time interval $T_i$ will be

$$R_i(x) = exp\left\{-\lambda\theta^{i-1\left[(x+qt_{i-1})^{\beta}-(qt_{i-1})^{\beta}\right]}\right\}, \quad 0 \le x \le T_i \quad (7)$$

and $$h_i(x) = \lambda\beta\theta^{i-1}(x + qt_{i-1})^{\beta-1} \quad 0 \le x \le T_i \quad (8)$$

respectively. If the failure rate is used to express the reliability function, then reliability function is expressed as

$$R_i(x) = exp\left[-\left(\int_0^x h_i\,(x)dx\right)\right] = exp\left[-\left(\int_0^x \lambda\beta\theta^{i-1}\,(x + qt_{i-1})^{\beta-1}dx\right)\right]\ 0 \le x \le T_i \quad (9)$$

In practice, the degree of each preventive maintenance action depend on the available resources, i.e., material, technological, human, time etc. [10] extended the improvement factor *q* and observed that it is a random variable with a value between *0* and *1*. [30]pointed out that the failure rate increase factor $\theta$ is not a fixed constant but usually falls within an interval.

In this particular case, it is assumed that the improvement factor $q(0 \le q \le 1)$ and the failure rate increase factor $\theta(u \ge \theta \ge 1)$ follow uniform distribution. Therefore, the corresponding $cdf_s$ are expressed by

$$F(q) = q, 0 \le q \le 1 \quad (10)$$

and $$F(\theta) = \frac{\theta-1}{u-1}, 1 \le \theta \le u \quad (11)$$

respectively, where *u* is a constant and the upper limit of the failure rate increase factor.

## 2.2 Derivation of preventive maintenance intervals

Integrating equation (8) with respect to *x* in the interval $[0, T_i]$, we get the number of failures $F_i$ for the $i_{th}$PM interval

$$F_i = \int_0^{T_i} h_i\,(x)dx$$
$$= \int_0^{T_i} \lambda\beta\theta^{i-1}(x + qt_{i-1})^{\beta-1}\,dx = \lambda T_1^{\beta}, i = 1$$
$$\int_0^{T_i} \lambda\beta\left(\int_1^u \theta dF(\theta)\right)^{i-1}\left(\int_0^1 (x + qt_{i-1})^{\beta-1}\,dF(q)\right)dx =$$
$$= \lambda\frac{u+1}{2}^{i-1}\frac{(T_i + t_{i-1})^{\beta+1} - t_{i-1}^{\beta+1} - T_i^{\beta+1}}{(\beta+1)t_{i-1}},$$
$$0 \le x \le T_i, i = 2,3,\dots,N \quad (12)$$

Assuming that an imperfect preventive maintenance is carried out once the system's reliability falls to the predetermined minimum value *R*. From equation (9), the system's reliability is expressed as

$$exp\left[-\left(\int_0^{T_i} \lambda\beta\theta^{i-1}(x + qt_{i-1})^{\beta-1}\,dx\right)\right] = R, i = 1,2,\dots,N \quad (13)$$

Taking natural logarithm of both sides of equation (13) leads to

$$\int_0^{T_i} \lambda\beta\theta^{i-1}\,(x + qt_{i-1})^{\beta-1}dx = -lnR, i = 1,2,\dots,N \quad (14)$$

Therefore, using equations (12) and (14), we get

$$\begin{Bmatrix} \lambda T_1^{\beta} = -lnR, i = 1 \\ \frac{\lambda}{\beta+1}\left(\frac{u+1}{2}\right)^{i-1} \frac{T_i^{\beta+1}+t_{i-1}^{\beta+1}-(T_i+t_{i-1})^{\beta+1}}{t_{i-1}} = lnR, i = 2,3,\dots,N \end{Bmatrix} \quad (15)$$

The sequence time intervals$T_i$, $(i = 1,2,\dots,N)$ can be determined iteratively using equation (15) as follows

$$\begin{Bmatrix} T_1 = \left(-ln\frac{R}{\lambda}\right)^{1/\beta}, i = 1 \\ T_i^{(k+1)} = \begin{Bmatrix} \left(T_i^{(k)} + t_{i-1}\right)^{\beta+1} - \\ t_{i-1}^{\beta+1} + \frac{(\beta+1)lnR}{\lambda}\left(\frac{2}{u+1}\right)^{i-1} t_{i-1} \end{Bmatrix}^{1/\beta+1} \\ i = 2,3,\dots,N \end{Bmatrix} \quad (16)$$

where $T_i^{(k+1)}$ *and* $T_i^{(k)}$ are *(k+1)$_{th}$* and $k_{th}$ iteration results respectively

1.2 Optimal values of *N* and *C(N)*

According to [5],the long run average cost rate *C(N)* with *N* cycles is

$$C(N) = \frac{C_m \sum_{i=1}^{N} F_i + (N-1)C_p + C_r}{\sum_{i=1}^{N} T_i}$$

$$= \frac{C_m \sum_{i=1}^{N} \int_0^{T_i} h_i(x)dx + (N-1)C_p + C_r}{\sum_{i=1}^{N} T_i} \quad (17)$$

where$C_m$, $C_p$ *and* $C_r$ are the costs for minimal repair, PM and system replacement respectively

Substituting $F_i$ from equation (12) into equation (17) yields

$$C(N) = \begin{Bmatrix} \frac{C_r - C_m\, lnR}{\left(-ln^{R}/_{\lambda}\right)^{1/\beta}}, i = 1 \\ \frac{C_m \lambda}{\beta+1}\sum_{i=2}^{N}\left[\left(\frac{u+1}{2}\right)^{i-1}\frac{(T_i+t_{i-1})^{\beta+1}-t_{i-1}^{\beta+1}-T_i^{\beta+1}}{t_{i-1}}\right], \\ \frac{-C_m\, lnR + (N-1)C_p + C_r}{\left(-\frac{lnR}{\lambda}\right)^{\frac{1}{\beta}} + \sum_{i=2}^{N} T_i} \\ i = 2,3,\dots,N \end{Bmatrix} \quad (18)$$

From equation (18), *C(N)* is a function of only *N* , therefore the optimal values *N** of *N* and *C(N*)* of *C(N)* can be found by substituting *N** into equation (18)

The optimal value *N** must satisfy the two inequalities $C(N+1) \geq C(N)$ and $C(N) <$ *CN−1* which implies that

$$\sum_{i=1}^{N} T_i \int_0^{T_{N-1}} h_{N+1}(x)dx - T_{N+1}\sum_{i=1}^{N}\int_0^{T_i} h_i(x)\, dx \geq \frac{T_{N+1}C_r - \left[\sum_{i=1}^{N} T_i - (N-1)T_{N-1}\right]C_p}{C_m} \quad (19)$$

and $\frac{T_N C_r - \left[\sum_{i=1}^{N-1} T_i - (N-1)T_{N+1}\right]C_p}{C_m}$

$$< \sum_{i=1}^{N-1} T_i \int_0^{T_N} h_N(x)dx - T_N \sum_{i=1}^{N-1}\int_0^{T_i} h_i(x)dx \quad (20)$$

Let

$$L(N) = \sum_{i=1}^{N} T_i \int_0^{T_{N+1}} h_{N+1}(x)dx - T_{N+1}\sum_{i=1}^{N}\int_0^{T_i} h_i(x)dx$$

and$B(i) = \int_0^{T_i} h_i(x)dx$

then, subtracting the right-hand side of inequality (20) from the left-hand side of inequality (19), we get

$$L(N) - L(N-1 = \sum_{i=1}^{N} T_i\, B(N+1) - T_{N+1}\sum_{i=1}^{N} B(i) - \sum_{i=1}^{N-1} T_i\, B(N) + T_N \sum_{i=1}^{N-1} B(i)$$
$$= [B(N+1) - B(N)]\sum_{i=1}^{N-1} T_i + (T_N - T_{N+1})\sum_{i=1}^{N-1} B(i) + [T_N B(N+1) - T_{N+1}B(N)] \quad (21)$$

From equation (14),

$$T_i = \left[(qt_{i-1})^{\beta} - \frac{lnR}{\lambda\theta^{i-1}}\right]^{1/\beta} - qt_{i-1}, i = 2,3,\dots,N \quad (22)$$

Since $\lim_{i\to\infty}\theta^{i-1} = \infty(\theta > 1)$ thus $\lim_{i\to\infty} T_i = 0$ . This implies that $T_i$ is a strictly decreasing function of $N$ and $T_i > T_{i+1}$. Conversely, from equation (14), $B(N+1) - B(N) = 0$. Consequently, the last term of equation (21)

$$T_N B(N+1) - T_{N+1}B(N) > T_{N+1}B(N+1) - T_{N+1}B(N) = = T_{N+1}[B(N+1) - B(N)] = 0$$

Therefore, $L(N) - L(N-1) > 0$. This implies that $L(N)$ is strictly increasing in $N$ and tends to ∞ as $N\to\infty$. Therefore, based on [4], when the hazard rate $h(t)$ is a continuous and strictly increasing function, then there exists a finite and unique *N** which satisfies the inequalities (21) and (22).

## III. Numerical Example

This section presents a numerical example to illustrate the proposed maintenance model.

A piece of manufacturing equipment is observed to deteriorate with increased usage and age, undergoes minimal repair upon failures between two adjacent PM actions and follows the Weibull failure process. The maximum likelihood estimates of β and λ are [34]

$$\beta = \frac{n}{\sum_{j=1}^{n} ln\frac{X_n}{X_j}}, \qquad \lambda = \frac{n}{X_n^{\beta}} \quad (23)$$

Using historical maintenance dataof the equipment under consideration, the input parameters are given as follows: $\beta$ = 1.4753,$\lambda$ = 0.00003, $R$ = 0.7, $u$ = 1.1, $C_p$ = 15000, $C_m$= 30000, $C_r$ = 900 000.

Table 1 gives the preventive maintenance time intervals and it is observed that the PM time intervals gradually decrease along with the increase in maintenance actions. This implies that the system is subject to degradation with usage and age.

Table 1 PM time intervals

| i | $T_i$ | ti |
|---|---|---|
| 1 | 578.43 | 578.43 |
| 2 | 411.04 | 989.47 |
| 3 | 342.53 | 1332.00 |
| 4 | 297.86 | 1629.86 |
| 5 | 268.34 | 1898.20 |
| 6 | 220.28 | 2139.95 |
| 7 | 202.15 | 2360.23 |
| 8 | 186.51 | 2562.38 |
| 9 | 172.78 | 2748.89 |
| 10 | 160.61 | 2921.67 |
| 11 | 149.69 | 3082.28 |
| 12 | 139.84 | 3231.97 |
| 13 | 130.89 | 3371.81 |
| 14 | 122.71 | 3502.70 |
| 15 | 115.21 | 3525.41 |
| 16 | 108.30 | 3740.62 |
| 17 | 101.92 | 3848.92 |
| 18 | 96.00 | 3950.84 |
| 19 | 90.50 | 4046.84 |
| 20 | 85.39 | 4137.84 |
| 21 | 80.62 | 4222.73 |
| 22 | 76.34 | 4303.35 |
| 23 | 72.02 | 4379.69 |

Figure 1 shows the failure rate of the imperfect sequential preventive maintenance policy. It can be observed that the equipment failure rate decrease after PM actions between the initial and 7$^{th}$ cycles but increases thereafter. This is due to the fact that, equipment is likely to suffer failures during the early stages of exploitation (infant mortality) due to manufacturing errors. The corrective maintenance actions are basically meant to correct such errors. The imperfect PM that follows thereafter reduces the equipment's effective age to a certain value rather than to zero. Since the failure rate is a function of the effective age and equipment usage, the initial failure rate value right after the PM action is not equivalent to zero. Imperfect PM changes the slope of the failure rate function and makes it more and more high due to deterioration process. When the failure rate increases and its function exceeds the ratio of the failure rate value just before PM to the equipment's effective age right after PM, then the failure rate right after each PM is increasing rather than decreasing at the corresponding PM time. Therefore, PM reduces the failure rate at first and then increases it.

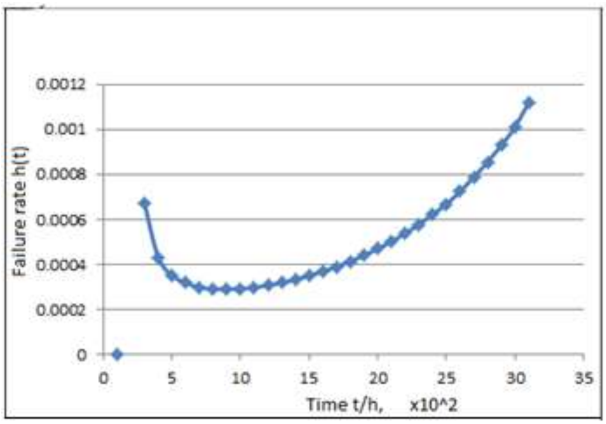


Fig. 1 Failure rate of PM

## IV. Sensitivity Analysis and Discussion

From equation (18) it is evident that the long-run average cost rate is affected by the five cost parameters $C_m$,$C_p$, $C_r$, *R* and *u* when β and λ are known. For analytical purposes, cost ratios $C_m/C_p$ ,$C_r/C_p$ are used. To show how the optimal maintenance policy depend on the different cost parameters, only one cost parameter is changed at a time while others remain constant. The optimal results are shown in Tables 2 to 5 respectively. The initial input parameters are highlighted in the tables.

### 4.1 Effect of *u*

Table 2 and Fig.2 show the optimal maintenance number *N** and its corresponding long-run cost *C(N*)*. The optimal maintenance number *N**decreases as *u*and the long run cost *C(N*)* increases. Therefore,during equipment wear period it is necessary to reduce maintenance actions before replacement or overhaul.

Table2 Optimum *N**,$T_N^*$ and *C(N*)* for different*u*

| **U=1.1** | U=1.2 | U=1.3 |
|---|---|---|
| **N* $T_N^*$ C(N*)** | N* $T_N^*$ C(N*) | N* $T_N^*$ C(N*) |
| **20 90.5 297.06** | 15 122.71 318.46 | 13 139.84 338.32 |

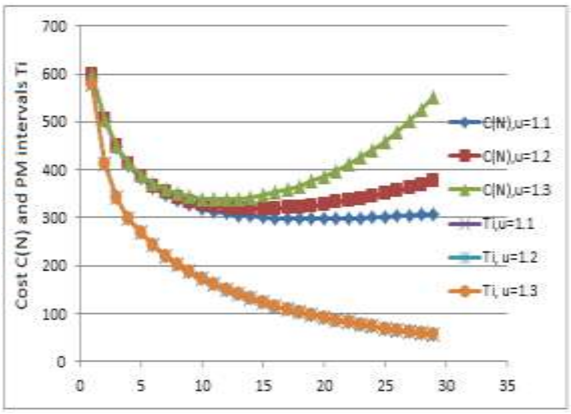


Fig.2 Long-run cost and PM intervals and number of PM actions with different *u*

### 4.2 Effect of *R*

Table 3 and Fig.3 shows that the optimal maintenance number *N** decrease and the corresponding long-run cost *C(N*)* increase with an increase of *R*. With higher reliability, the long-run cost of frequent maintenance is higher whereas the length of time interval between PM actions becomes shorter and shorter. This implies that, it is worthwhile and necessary to increase the number of maintenance actions for equipment with a higher reliability requirement.

Table3. Optimum N*, $T_N^*$, and C(N*) for different *R*

| R=0.6 | **R=0.7** | R=0.8 |
|---|---|---|
| N* $T_N$* C(N*) | **N* $T_N$* C(N*)** | N* $T_N$* C(N*) |
| 18 128.8 248.7 | **20 90.5 297.1** | 22 58.13 385.3 |

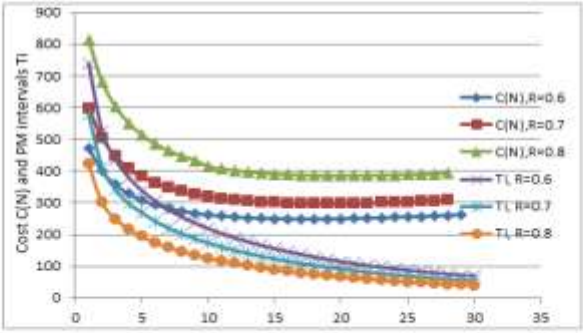


Fig.3 Long-run cost and PM intervals and number of PM actions with different *R*

### 4.3 Effect of $C_m/C_p$

Table 4 Optimum *N** and *C(N*)* for different $C_m/C_p$

| $C_m/C_p$ | 0.5 1.0 1.5 **2.0**2.5 3.0 5.0 10.0 |
|---|---|
| N* | 25 23 21 **20** 19 18 15 11 |
| C(N*) | 259.5 273.1 285.4 **297.1** 308.4 319.3 360.2 448.6 |

From Table 4, as the cost ratio $C_m/C_p$ increases, the optimal maintenance number N* decreases while the corresponding long-run cost *C(N*)* increases which implies that, the higher the cost ratio $C_m/C_p$, the less the cost-effectiveness PM work is.

### 4.4 Effect of $C_r/C_p$

Table 5 gives the results of the optimum *N** and *C(N*)* for different $C_r/C_p$. It is observed that both the optimal maintenance number *N** and the corresponding long-run cost *C(N*)* increase as the cost ratio $C_r/C_p$ increase.

Generally, the more expensive the equipment is, the more maintenance actions are required and the higher is the cost.

Table 5 Optimum *N** and *C(N*)* for different $C_r/C_p$

| $C_r/C_p$ | 10 | 20 | 40 | **60** | 80 | 100 |
|---|---|---|---|---|---|---|
| $N^*$ | 5 | 10 | 16 | **20** | 24 | 26 |
| $C(N^*)$ | 106.42 | 155.03 | 230.97 | **297.06** | 358.68 | 417.52 |

## V. Conclusions

A sequential imperfect maintenance policy for deteriorating system with variable maintenance quality considering reliability limit is presented. In practice, it is usually difficult to determine the quality of maintenance actions as each sequential maintenance action has a different maintenance quality due to the prevailing operating conditions and available resources. The optimal maintenance policy is optimized by assuming that the failure rate increase factor and the restoration factor are both random variables with uniform probability distribution and that, the equipment under consideration obeys weibull process. The sensitivity analysis shows that in order to achieve optimal practical requirements of high reliability, it is necessary to consider either system's reliability limit or failure rate.

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